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Taper and stem outside-bark volume equations for five pine species in mixed-species forests in Mexico

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Introduction

Taper and volume equations are basic components of stand inventory, growth and yield prediction, forest planning, and product simulation systems. Ten taper equations derived from upper-height based on volume ratio functions were compared with a segmented compatible taper-volume equation. These systems were simultaneously fitted to taper and cumulative volume data for Arizona pine (Ap: *Pinus arizonica* Engelm.), Aztec pine (Azp: *P. teocote* Schiede ex Schltdl. & Cham.), Durango pine (Dp: *P. durangensis* Martínez), Mexican white pine (Mwp: *P. ayacahu*ite Ehrenb. ex Schltdl.), and Smooth-leaves pine (Slp: *P. leiophylla* Schiede ex Schltdl. & Cham.) in mixed-species forests in Mexico.

Materials and methods

Study area and data

The study area is called "Ejido San Diego de Tezains". It is located in mixed-species forests in Durango, Mexico. Total forest area is 61,098 ha, of which 28,636 ha are for timber production.

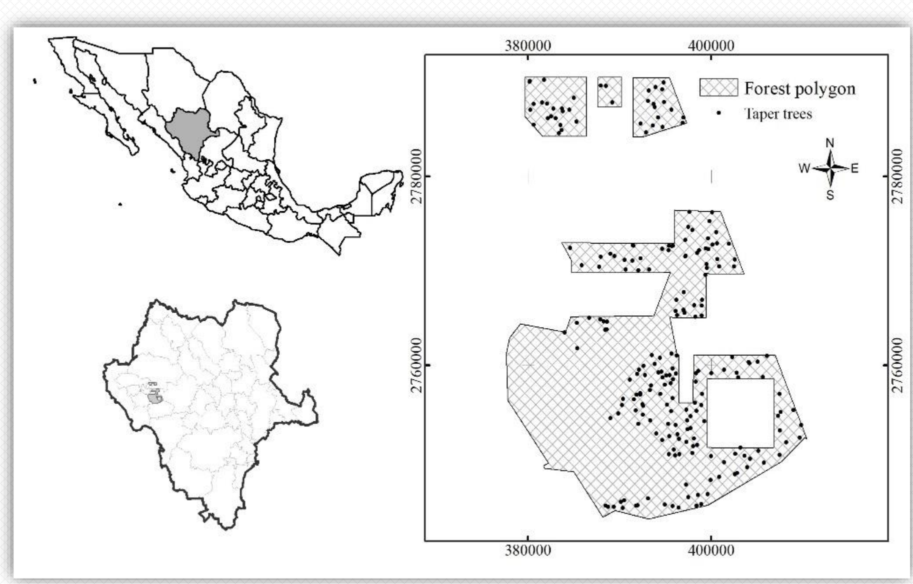

Figure 1. Study area and locations of the sampled trees in Durango Mexico.

The taper data consist of 46, 51, 69, 26 and 30 trees of Ap, Azp, Dp, Mwp and Slp, respectively.

Systems of taper and stem volume equations

Ten upper-height based volume ratio equations and corresponding compatible taper functions (Zhao & Kane, 2017; Lynch et al. 2017), and the segmented compatible taper-volume system proposed by Fang et al. (2000) (Table 1) were fitted and compared.

All systems were simultaneously fitted to taper measurements and cumulative stem outside-bark volume data for each species with weighted nonlinear seemingly unreleated regression (NSUR) method.

Then, the compatible systems were ranked based on fitting statistics, overall biases and SEEs of diameter and volume, and the biases and SEE of diameters and volumes for different proportions of the stem, using a ranking system of Kozak & Smith (1993).

Table 1: Taper and stem merchantable volume equations

System	Variable-top equation	Taper equation
CRS1	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}\left[1-(1-p)^{\beta_1}\right]^{\beta_2}$	$d(h)=\sqrt{\frac{\alpha_0 D^{\alpha_1}H^{\alpha_2}\beta_1\beta_2}{H}[1-(1-p)^{\beta_1}]^{\beta_2-1}(1-p)^{\beta_1-1}}$
CRS2	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}\left[1-(1-p)^{\beta_1}\right]^{\{1-\beta_2 exp[-exp(\beta_3 D^{\alpha_1}H^{\alpha_2})]\}}$	$d(h)=\sqrt{\frac{\beta_1}{kH}\alpha_0 D^{\alpha_1}H^{\alpha_2}(1-p)^{\beta_1-1}\{1-\beta_2 exp[-exp(\beta_3 D^{\alpha_1}H^{\alpha_2})]\}[1-(1-p)^{\beta_1}]^{\{-\beta_2 exp[-exp(\beta_3 D^{\alpha_1}H^{\alpha_2})]\}}}$
CRS3	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}\left[1-(1-p)^{\beta_1}\right]$	$d(h)=\sqrt{\frac{\beta_1\alpha_0 D^{\alpha_1}H^{\alpha_2}}{Hk}(1-p)^{\beta_1-1}}$
CRS4	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}\left[1-(1-p)exp(-\beta_1 p)\right]$	$d(h)=\sqrt{\frac{\alpha_0 D^{\alpha_1}H^{\alpha_2}}{Hk}exp(-\beta_1 p)(1+\beta_1-\beta_1 p)}$
CRS5	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}\{p\,exp[\beta_1(1-p)]\}$	$d(h)=\sqrt{\frac{\alpha_0 D^{\alpha_1}H^{\alpha_2}}{Hk}exp[\beta_1(1-p)](1-\beta_1 p)}$
CRS6	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}\left[1-(1-p)^{\beta_1}exp(-\beta_2 p)\right]$	$d(h)=\sqrt{\frac{\alpha_0 D^{\alpha_1}H^{\alpha_2}}{Hk}exp(-\beta_2 p)(1-p)^{\beta_1-1}[\beta_1+\beta_2(1-p)]}$
CRS7	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}\left[1-(1-p)\frac{\beta_1}{\beta_1+p}\right]$	$d(h)=\sqrt{\frac{\alpha_0 D^{\alpha_1}H^{\alpha_2}(\beta_1^2+\beta_1)}{Hk(\beta_1+p)^2}}$
CRS8	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}\left\{1+\beta_1\left[\frac{(H-h)^{\beta_2}}{H^{\beta_3}}\right]\right\}$	$d(h)=\sqrt{-\frac{\alpha_0 D^{\alpha_1}H^{\alpha_2}}{k}\beta_1\beta_2\left[\frac{(H-h)^{\beta_2-1}}{H^{\beta_3}}\right]}$
CRS9	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}\left[1-\beta_1(1-p)^{\beta_2}\right]$	$d(h)=\sqrt{\alpha_0 D^{\alpha_1}H^{\alpha_2}\frac{\beta_1\beta_2}{kH}(1-p)^{\beta_2-1}}$
CRS10	$V(h)=\alpha_0 D^{\alpha_1}H^{\alpha_2}-\beta_2 D^{\beta_3}H^{\beta_4}(1-p)^{\beta_1}$	$d(h)=\sqrt{\frac{\beta_1}{kH}\beta_2 D^{\beta_3}H^{\beta_4}(1-p)^{\beta_1-1}}$
CSS	$V(h)=c_1^2 H^{(k/\beta_1)}[\beta_1 t_0+(I_1+I_2)(\beta_2-\beta_1)t_1+I_2(\beta_3-\beta_2)A_1 t_2-R(1-p)^{k/R}A_1^{I_1+I_2}A_2^{I_2}]$ With $c_1=\sqrt{\alpha_0 D^{\alpha_1}H_i^{(\alpha_2-K)/\beta_1}/[\beta_1(t_0-t_1)+\beta_2(t_1-A_1 t_2)+\beta_3 A_1 t_2]}$ $t_0=(1-p_0)^{K/\beta_1};\ p_0=h_b/H;\ t_1=(1-\vartheta_1)^{K/\beta_1};\ t_2=(1-\vartheta_2)^{K/\beta_2}$ $A_1=(1-\vartheta_1)^{(\beta_2-\beta_1)K/\beta_1\beta_2};\ A_2=(1-\vartheta_2)^{(\beta_3-\beta_2)K/\beta_2\beta_3};\ R=\beta_1^{1-(I_1+I_2)}\beta_2^{I_1}\beta_3^{I_2}$ $I_1=\begin{cases}1 & si\ \vartheta_1\le\left(\frac{h}{H}\right)\le\vartheta_2\\0 & Otherwise\end{cases};\ I_2=\begin{cases}1 & si\ \vartheta_2<\left(\frac{h}{H}\right)<1\\0 & Otherwise\end{cases}$ $\vartheta_1=h_1/H;\ \vartheta_2=h_2/H$	$d(h)=c_1\sqrt{\left[H^{(k-\beta_1)/\beta_1}(1-p)^{(k-R)/R}A_1^{(I_1+I_2)}A_2^{I_2}\right]}$

V(h): merchantable stem outside-bark volume (m^3) at upper-stem height h (m); d(h): outside-bark diameter (cm) at upper-stem height h (m); H: total tree height (m); p: relative height (p =h/H); D: diameter at breast height (cm); k = π/40,000; α_i and β_i: parameters to be estimated.

Results and discussion

Table 2: Sum of ranks and ranks of the eleven systems by species

Species	CRS1	CRS2	CRS3	CRS4	CRS5	CRS6	CRS7	CRS8	CRS9	CRS10	CSS
Ap	265 (2)	258 (1)	316 (7)	320 (8)	294 (3)	309 (5)	363 (11)	346 (10)	310 (6)	340 (9)	301 (4)
Azp	273 (2)	248 (1)	303 (5)	339 (7)	298 (3)	311 (6)	362 (9)	339 (7)	311 (6)	346 (8)	302 (4)
Dp	274 (2)	251 (1)	302 (5)	345 (8)	291 (3)	311 (6)	364 (10)	330 (7)	311 (6)	347 (9)	296 (4)
Mwp	229 (1)	261 (2)	334 (9)	298 (4)	293 (3)	320 (6)	332 (8)	358 (10)	321 (7)	360 (11)	308 (5)
Slp	268 (2)	261 (1)	305 (5)	354 (10)	292 (3)	314 (7)	383 (11)	328 (8)	306 (6)	331 (9)	302 (4)

Table 3: Fitting statistics for the best one system for each species

Species	System	*d (cm)*						*Vh (m^3)*					
		R^2	*SEE*	*% SEE*	*Bias*	*% bias*	*AIC*	*R^2*	*SEE*	*% SEE*	*Bias*	*% bias*	*AIC*
Ap	CRS2	0.98	1.4	7.1	-0.17	-0.9	607	0.98	0.07	11.8	-0.003	-0.6	-4292
Azp	CRS2	0.98	1.5	7.5	-0.17	-1.1	620	0.99	0.03	8.3	-0.001	-0.2	-5281
Dp	CRS2	0.98	1.4	7.4	-0.11	-0.6	791	0.99	0.03	9.8	-0.001	-0.1	-7017
Mwp	CRS1	0.98	1.5	7.1	-0.01	-0.2	343	0.99	0.05	9.8	0.002	0.3	-2455
Slp	CRS2	0.98	1.3	6.4	-0.18	-0.8	292	0.99	0.03	6.0	-0.003	-0.5	-3412

The CRS2 and CSS were best for diameter predictions, and the CRS5 and CRS3 were best for merchantable and total volume predictions for all five pine species. The CRS2 system was the best for Ap, Azp, Dp and Slp species, while the CRS1 performed best for Mwp.

Conclusions

All top three systems of taper and volume equations derived from upper-height-based volume ratio functions are algebraically compatible, and outperformed the segmented-stem compatible taper-volume system (Fang et al. 2000) in terms of taper and volume predictions.

Literature cited

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